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POWER LOSSES IN LIQUID METAL CURRENT COLLECTORS.(U)  
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**DAVID W. TAYLOR NAVAL SHIP  
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POWER LOSSES IN LIQUID METAL  
CURRENT COLLECTORS

Myles M. Hurwitz and Dolores R. Wallace

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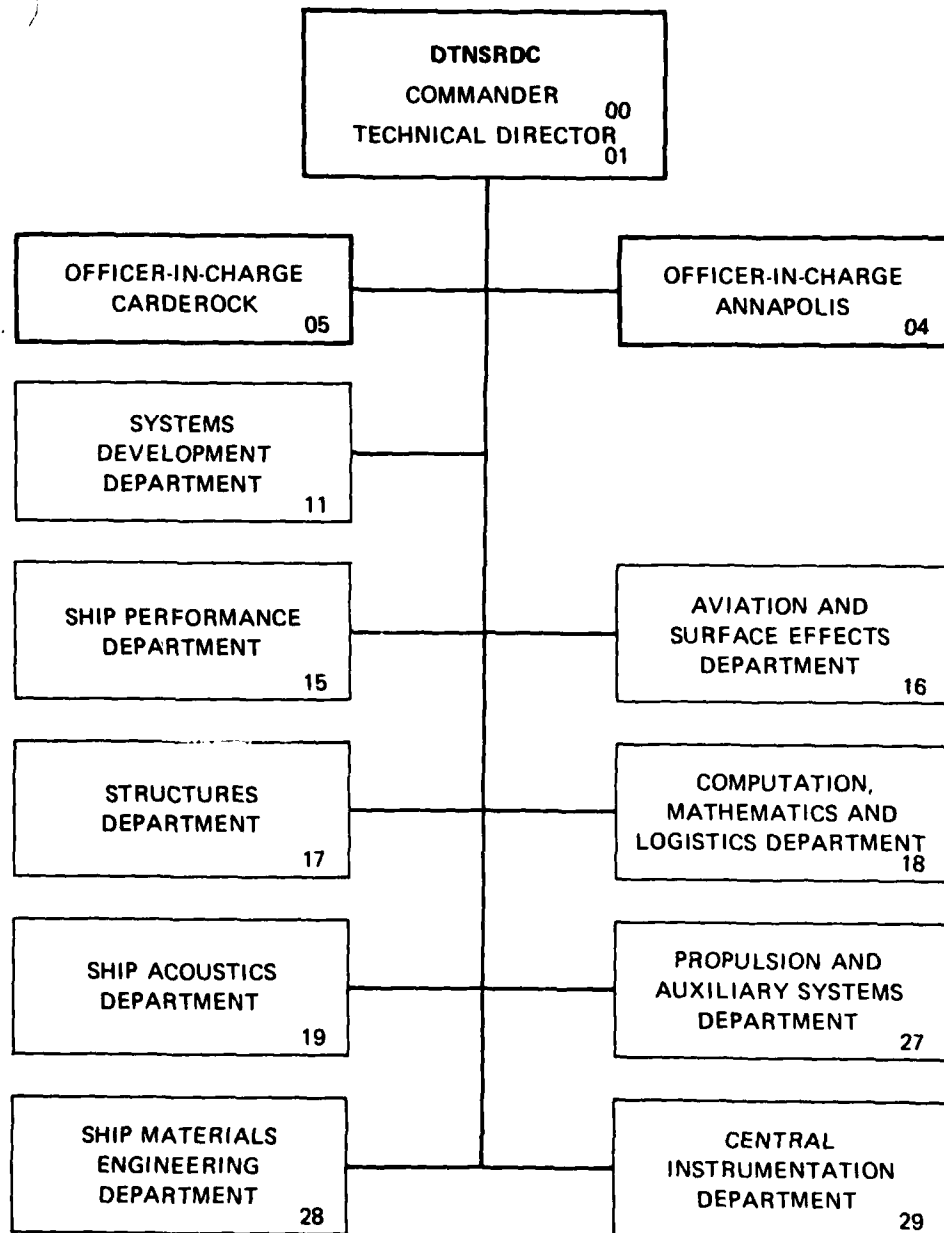
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## TABLE OF CONTENTS

	Page
LIST OF FIGURES.....	iii
ABSTRACT.....	1
ADMINISTRATIVE INFORMATION.....	1
INTRODUCTION.....	1
GENERAL SOLUTION OUTLINE.....	4
RHODENIZER'S EXPRESSIONS.....	4
NEW EXPRESSIONS.....	6
METHOD OF SOLUTION.....	9
DISCUSSION.....	10
COMPARISON WITH EXPERIMENT.....	11
ACKNOWLEDGMENTS.....	13
REFERENCES.....	13
APPENDIX - DESCRIPTION OF THE PWRLOSS PROGRAM.....	15

## LIST OF FIGURES

1 - Collector Geometry and Nomenclature.....	2
2 - Rotor and Channel Geometries.....	3
3 - Semi-infinite Collector Geometry.....	5
4 - Circulating Current Geometry.....	7
5 - Finite Element Model.....	8
6 - Braid Holder Fit in Stator.....	11

## ABSTRACT

A numerical capability has been developed which will compute ohmic and viscous power losses in liquid metal current collectors. The present work extends previous analytical investigations in that semi-infinite collector geometries are no longer assumed. This new capability is based on the finite element method and makes use of electrical current densities computed by the heat transfer portion of the NASTRAN structural analysis program. Although some limitations and questions remain, a comparison between the new numerical capability and experiment shows very good agreement in the computation of the power losses.

## ADMINISTRATIVE INFORMATION

The work reported was performed under Task Area S0380SL001, Task 16761, Work Unit 1-2706-100.

## INTRODUCTION

The Superconductive Machinery Development effort at DTNSRDC involves investigation into the design and operation of motors and generators which require current collecting systems capable of high-current, low-voltage performance. One such system involves the containment of a liquid metal in a tongue and groove collector (Figure 1). A rotor tongue carries the liquid metal around the collector, thereby allowing transfer of electrical current across the groove. The most active candidate for the liquid metal is a low density, highly conductive eutectic sodium potassium alloy, NaK (22% Na, 78% K). Requirements, problems, and possible collector shapes (Figure 2) were discussed in a report<sup>1\*</sup> documenting progress on liquid metal current collector development. In the past, these areas have been addressed mostly by testing. Work by Rhodenizer<sup>2,3</sup> on treating these areas analytically assumed a semi-infinite collector geometry. The present project expands on Rhodenizer's work by removing the restriction of a semi-infinite geometry.

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\*A complete listing of references is given on page 13.

$W_c$  = Collector width  
 $R_c$  = Collector radius  
 $A_c$  = Collector area  
 $G_{ca}$  = NaK axial gap  
 $G_{cr}$  = RaK radial gap  
 $W_d$  = Disk width  
 $X_d$  = Disk wetted sidewall  
 $A_{dw}$  = Disk wetted surface  
 $L_d$  = Disk length  
 $\bar{R}_d$  = Disk mean radius

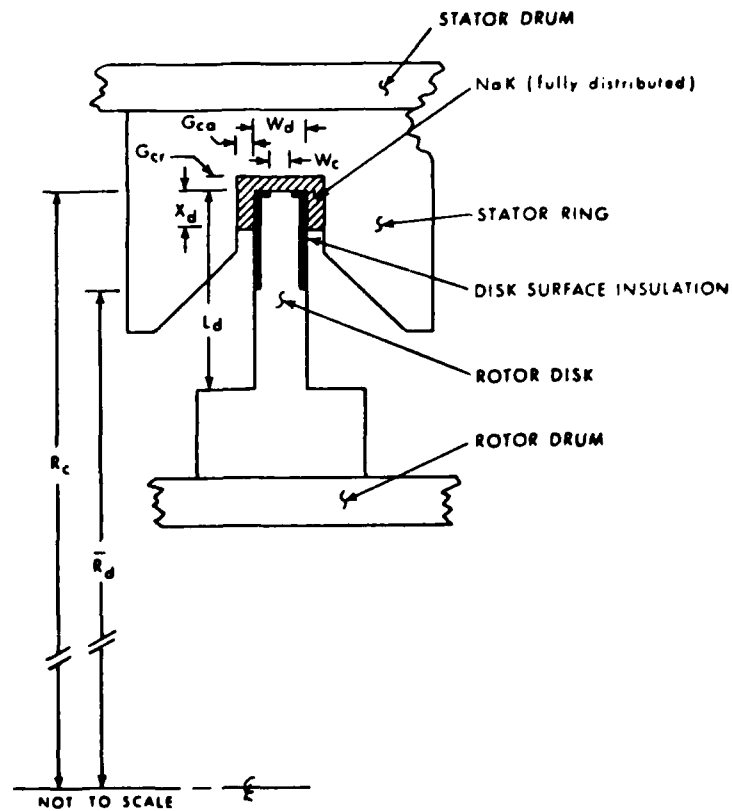


Figure 1 - Collector Geometry and Nomenclature



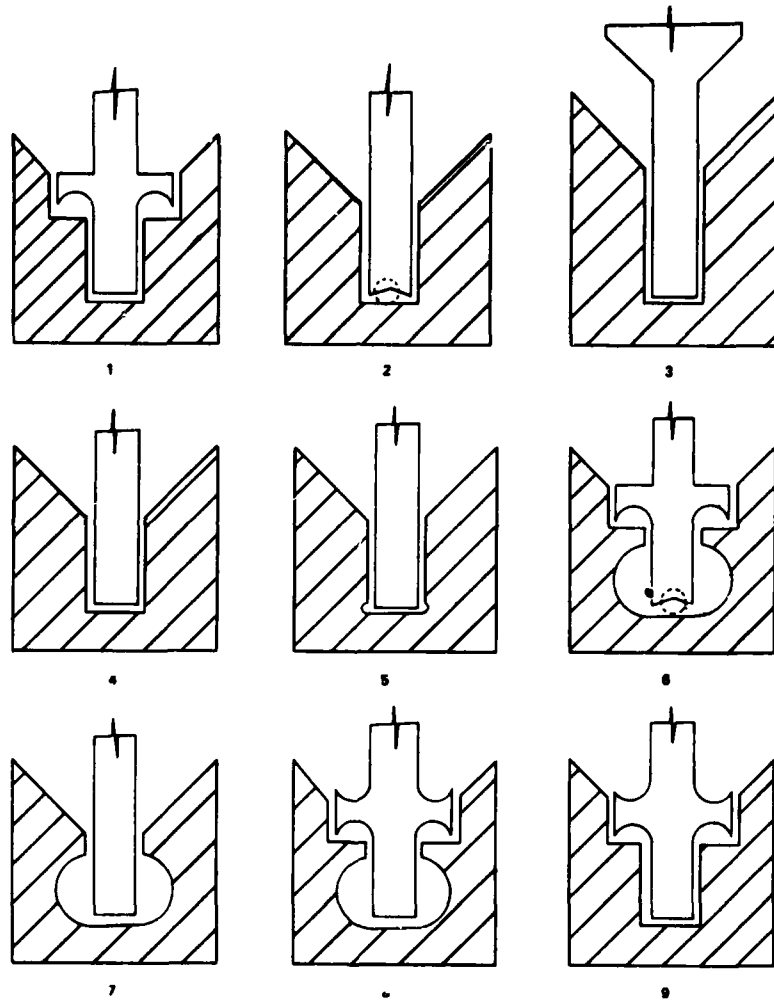


Figure 2 - Rotor and Channel Geometries

In order to represent the physical situation numerically, the flow of the liquid metal should be known. However, "the mathematical problem of describing the magnetohydrodynamic flow of a conducting liquid, carrying electric current in a magnetic field and with an external force causing fluid flow, is extremely complicated and, in fact, has not been completely solved."<sup>1</sup> Therefore, the goal of the present effort is to remove some of Rhodenizer's limiting assumptions and then to predict power losses in a liquid metal current collector by numerical methods.

#### GENERAL SOLUTION OUTLINE

Rhodenizer's assumptions of semi-infinite collector geometry and uniform radial transport current were removed by using finite elements to approximate the liquid metal filling an assumed collector geometry. The finite elements used were the quadrilateral membrane (QDMEM) elements of the NASTRAN structural analysis computer program.<sup>4</sup> While NASTRAN is primarily a structural analysis program, it does contain an extensive heat transfer capability in which the Laplace and Poisson equations are solved. Also, computation of the distribution of steady-state electric potential is a direct analogy to the heat transfer computation. In addition to electric potential (temperature), NASTRAN also computes electric current density (heat flux) distribution. The current density within one QDMEM element is assumed constant.

Once the problem is modeled with finite elements, NASTRAN computes the distribution of transport current density in the collector, assuming zero potential on the stator and a nonzero potential across the rotor at a specified distance from the rotor tip. From these results, eddy current density and fluid velocity distributions are found. Finally, viscous and ohmic power losses are computed.

#### RHODENIZER'S EXPRESSIONS

The collector geometry assumed by Rhodenizer<sup>2,3</sup> is shown in Figure 3. Rhodenizer assumes uniform radial transport current density  $J_y$  through the gap  $d$ , with axial transport current density  $J_x$  zero. The

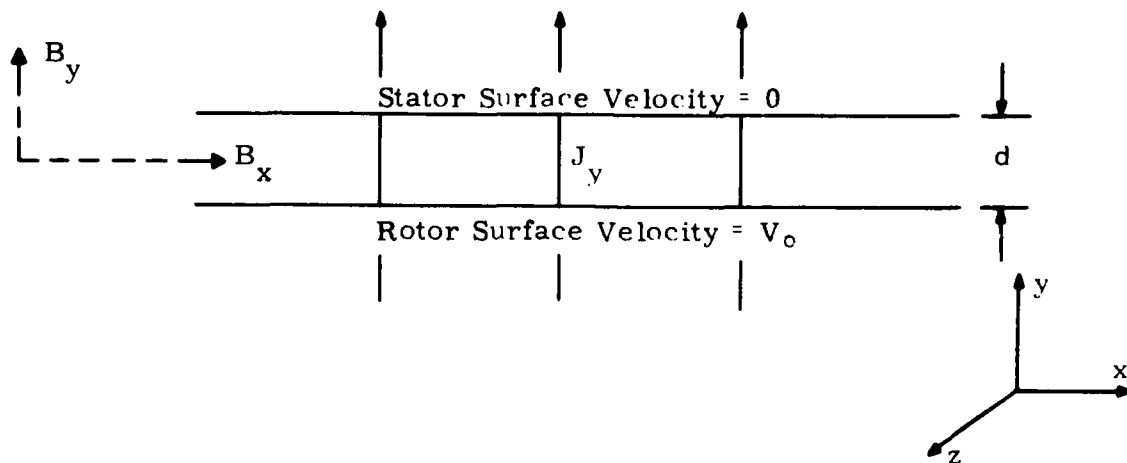


Figure 3 - Semi-infinite Collector Geometry

flow is assumed to be turbulent, and, therefore, uniform except at the rotor and stator boundaries. The fluid velocity is computed by either

$$V_f = \frac{V_0}{2} (1-\delta) , \delta \leq 1 \quad (1)$$

$$V_f = \frac{V_0}{2} (1-\sqrt{2\delta-1}) , \delta \geq 1 \quad (2)$$

or

$$V_f = \frac{V_0}{2} (1+\delta) , \delta \leq 1 \quad (3)$$

$$V_f = \frac{V_0}{2} (1+\sqrt{2\delta-1}) , \delta \geq 1 \quad (4)$$

where

$V_0$  = rotor tip velocity

$$\delta = \left| \frac{2 B_x J_y d}{f \rho V_0^2} \right| \quad (5)$$

$B_x$  = axial magnetic field induction

$f$  = Fanning friction factor

$\rho$  = NaK density

Equations (1) and (2) are used if  $\vec{J} \times \vec{B}$ , the Lorentz force density, has the opposite sign of  $V_0$ , which retards fluid velocity. Equations (3) and (4) are used if  $\vec{J} \times \vec{B}$  increases fluid velocity.

Once the fluid velocity has been computed, ohmic and viscous losses can be found by<sup>5</sup>:

$$P_{\text{ohmic}} = \frac{I^2}{2\pi r w} \left( \frac{d}{\sigma} + \epsilon \right) \quad (6)$$

where  $I$  = load current

$r$  = rotor radius

$w$  = collector electrical contact width

$\sigma$  = NaK electrical conductivity

$\epsilon$  = solid-liquid-solid specific contact resistance, measured as approximately  $1. \times 10^{-9}$  ohm -  $m^2$

and

$$P_{\text{viscous}} = \frac{\pi}{4} f \rho V_0^3 r w (1 + 3\delta^2) , \delta \leq 1 \quad (7a)$$

$$P_{\text{viscous}} = \frac{\pi}{2} f \rho V_0^3 r w (1 + \delta) (\sqrt{2\delta - 1}) , \delta \geq 1 \quad (7b)$$

Rhodenizer also considers the effect of an additional complicating factor: the presence of circulating currents produced when a moving conductor (NaK) cuts the flux of a magnetic field. With Rhodenizer's simplifying assumptions, he can limit these currents to closed loops about the center of the collector (Figure 4). In that case, the circulating current densities are a function of axial position (i.e.,  $j_y(x)$  can be computed by assuming an induced electromotance of zero) and should be taken into account when computing ohmic and viscous losses.

Rhodenizer admits that, when  $\delta$  (or  $\delta(x)$  when circulating currents are taken into account) varies through the collector ( $\delta > 1$  in some places,  $\delta < 1$  in others), closed-form solutions are not possible.

#### NEW EXPRESSIONS

The present work is based on the geometry given in Figure 1. A finite element model is shown in Figure 5. Disk surface insulation is

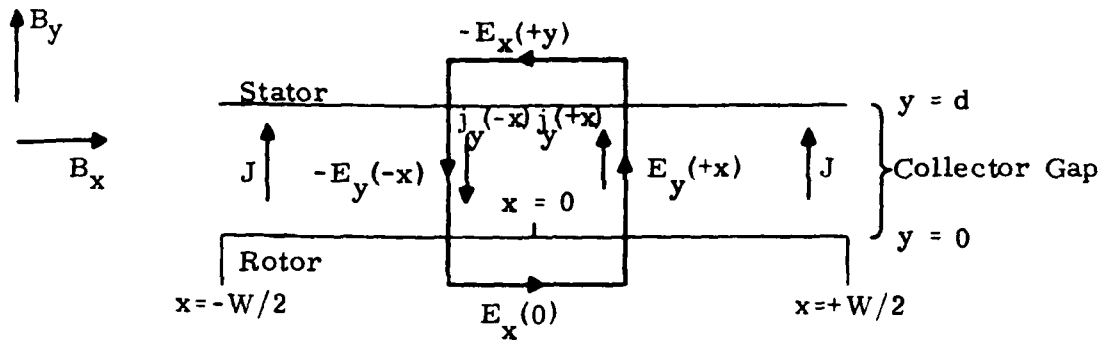


Figure 4 - Circulating Current Geometry

achieved by positioning coincident, but unconnected, grid points along the line of insulation. Because in the cases under consideration disk wetted sidewall length was set to zero and the NaK axial gap  $G_{ca}$  was very small, a simplifying assumption was made that the rotor width  $W_d$  was equal to the stator width. The expressions for fluid velocity  $V_f$  are the same as in Equations (1)-(4), but Equation (5) for  $\delta$  becomes

$$\delta(x,y) = \frac{2AN}{f\rho V_0^2 W_d} \{ B_x [J_y(x,y) + j_y(x,y)] - B_y [J_x(x,y) + j_x(x,y)] \} \quad (8)$$

where

$A$  = area of a finite element

$N$  = number of NaK finite elements in the mesh

$B_x, B_y$  = axial, radial components of magnetic field induction

$J_x, J_y$  = axial, radial components of transport current density

$j_x, j_y$  = axial, radial components of circulating current density

Once  $\delta(x,y)$  has been computed,  $V_f(x,y)$  can be found with Equations (1)-(4). Then the following expressions are used for ohmic and viscous losses:

$$P_{ohmic} = \sum_i \{ 2\pi r [(J_y + j_y)^2 \Delta x (\epsilon + \frac{\Delta y}{\sigma}) + (J_x + j_x)^2 \Delta y (\epsilon + \frac{\Delta x}{\sigma})] \} \quad (9)$$

where  $r$  is the radius of the centroid of the element and the sum is taken over all NaK finite elements.

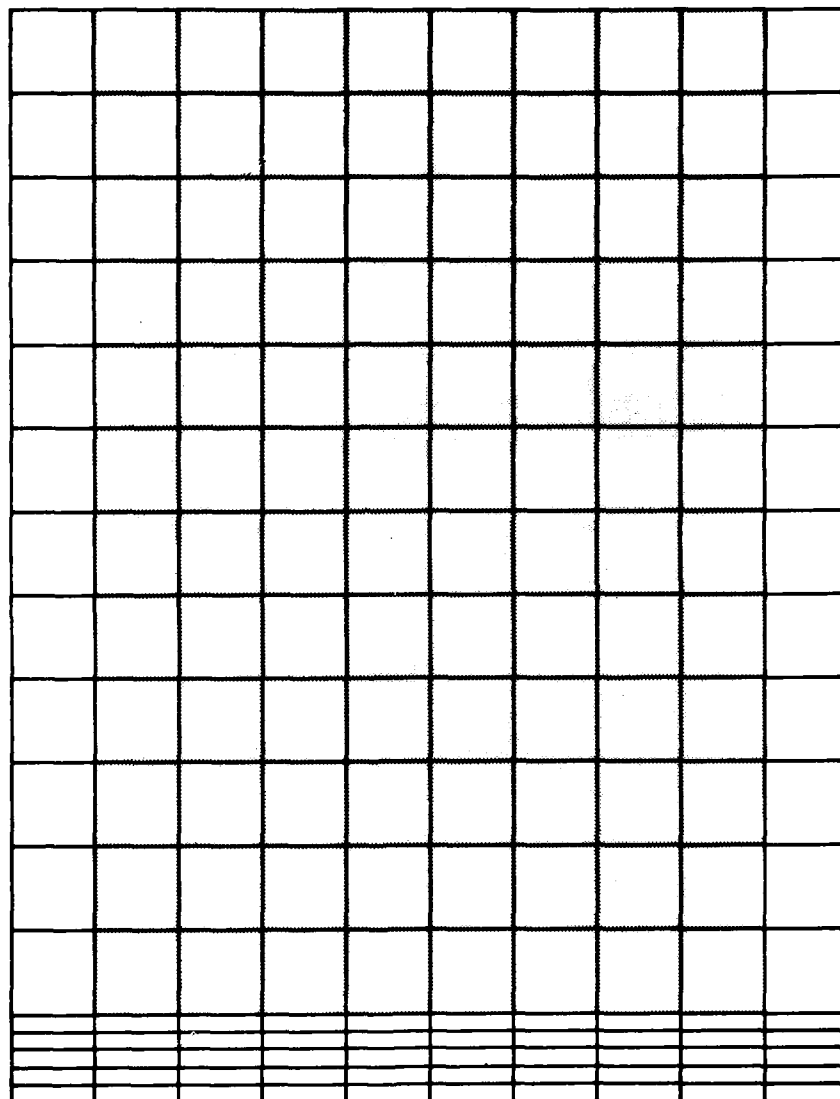


Figure 5 - Finite Element Model

$$\begin{aligned}
P_{\text{viscous}} = & \sum_i \frac{f\rho}{2} (V_0 - V_f)^2 V_0 [2\pi r_R \Delta x] \\
& - \sum_j [B_x (J_y + j_y) - B_y (J_x + j_x)] V_f [2\pi r_A]
\end{aligned}
\tag{10}$$

where

$r_R$  = rotor radius

$\Delta x$  = axial length of a finite element

$\sum_i$  = summation taken over NaK finite elements adjacent to  
rotor in NaK radial gap

$\sum_j$  = summation taken over all NaK finite elements

#### METHOD OF SOLUTION

The solution process proceeds as follows:

1. NASTRAN computes the transport current density  $\vec{J}$  for each QDMEM element and stores the result on a FORTRAN-readable file.
2. A new program PWRLOSS, described in the Appendix, was written to perform the remainder of the analysis. Its first step is to compute  $J_x$  and  $J_y$  at all mid-edges by averaging individual transport current densities from appropriate adjacent elements.
3. Each finite element is now considered to be an electrical circuit or loop, part of a network to which Kirchhoff's rules<sup>6</sup> will be applied. Each edge of the element is a conductor with an unknown current composed of two parts, the known transport current and the unknown circulating current. The induced electromotance,  $\oint \vec{E} \cdot d\vec{\ell} = \oint (\vec{V}_f \times \vec{B}) \cdot d\vec{\ell}$ , where  $\vec{\ell}$  is taken around the loop, is computed. Then  $\vec{V}_f$  is computed from Equations (1)-(4). ( $\vec{V}_f$  is uni-directional, i.e., z-direction only), with  $\delta$  coming from Equation (8). Note that  $\delta$  is unknown since it includes the unknown circulating current density. Also, if either Equation (2) or (4) is used, the resulting set of simultaneous equations, obtained by applying Kirchhoff's rules, will be nonlinear.
4. The Newton-Raphson method<sup>7</sup> is used to solve the set of simultaneous equations for the circulating currents.

5. Equations (1)-(4) and (8) are now used to compute the fluid velocity on each edge of the element. An average velocity for the element is computed.

6. Finally, Equations (9) and (10) are used to compute the ohmic and viscous power losses, respectively.

#### DISCUSSION

During the course of this project, a number of questions arose concerning the proper numerical handling of some of the physical aspects of the problem. Some of the questions related to a copper braid (Figure 6) inserted into the collector to help maintain the stability of the liquid metal.<sup>8</sup> This section discusses these questions, as follows:

1. Is the temperature of the rotor higher than that of the collectors, thus raising copper and machine resistance? We assume constant temperature.

2. Should copper conductivity or NaK conductivity (or something in between) be used to account for the NaK-filled braid? We used only the NaK conductivity.

3. The  $\epsilon$  term in Equations (6) and (9) is based on a loss resulting from a copper-NaK-copper interface. With the braid in place, how many interfaces must be accounted for? We assume one interface and a value for  $\epsilon$  of  $.65 \times 10^{-9}$  ohm-m<sup>2</sup>.

4. What is the size of the true gap? Should it be the rotor tip-to-braid gap or the rotor tip-to-stator gap, including NaK and braid as a part of the gap? We used the latter.

5. What is the effective conducting width of the collector tip as opposed to the geometrical width? We set the conducting width equal to the geometrical width.

6. What value should be used for the Fanning friction factor? Values from .0055 to .007 have been suggested. We used .0055.

7. If transport current densities  $J_x$  and  $J_y$  are assumed to exist on each element mid-edge, how do we treat  $J_x$  on vertical edges and  $J_y$  on horizontal edges, since the Kirchhoff loop assumes one-dimensional conductors? We used only  $J_x$  on horizontal edges and only  $J_y$  on vertical



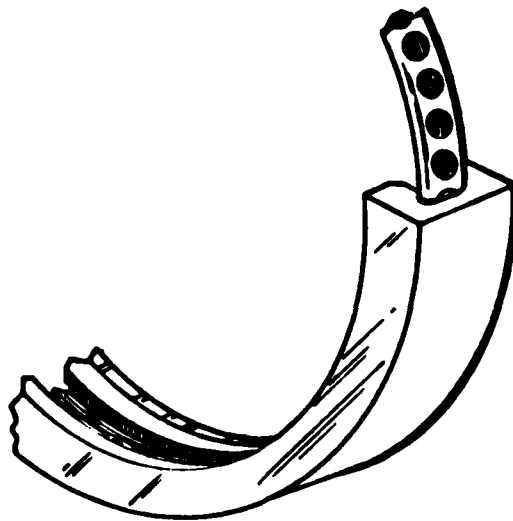


Figure 6 - Braid Holder Fit in Stator

edges.

8. How should we treat the velocity at an element edge which coincides with a stator or rotor edge? The velocity of such an element edge can be computed from Equations (1)-(4). However, although the velocities of the stator and rotor are 0 and  $V_0$ , respectively, these velocities should probably be treated as boundary layer effects. Therefore, to compute power losses, we used an edge velocity which is the average of the computed velocity and the constant velocity.

9. What value of NaK electrical conductivity should be used? Values range from  $2.2 \times 10^6$  mho/m to  $3.0 \times 10^6$  mho/m. We used  $3.0 \times 10^6$  mho/m.

It is expected that a number of these questions will be addressed in future experimental work.

#### COMPARISON WITH EXPERIMENT

A set of experiments was run with a generator system containing two liquid metal current collectors.<sup>8</sup> The average dimensions of the two collectors were:

collector radius $R_c$ :	.0665 m
collector width $w$ :	.0038 m
rotor tip-to-stator gap $d$ :	.0018 m
rotor tip velocity $V_0$ :	40 m/sec
generator current $I$ :	9498 A
axial magnetic field $B_x$ :	5T
radial magnetic field $B_y$ :	.1T

The specific experiment considered here was run 2 Nov 1978 at 1450 hours. The conditions of the experiment, e.g. generator current and voltage, are "believed to be accurate."<sup>8</sup> The total power loss in the test was measured by considering differential temperatures of the coolant. This measure is accurate if the generator had been running long enough at a particular power point to establish thermal equilibrium. This was assumed to be the case.

The total power loss measured, 2450 watts, is expected to be composed of four parts: ohmic loss through the rotor tip-to-stator gap, viscous loss, ohmic loss through the generator, and bearing and windage loss. The last two losses for this test were assumed to be 520 watts and 630 watts, respectively.

At this point, some preliminary computations will serve to show some of the numerical difficulties encountered. Equation (6) is used to compute the ohmic loss, with  $d = .0018\text{m}$ ,  $\sigma = 3.0 \times 10^6 \text{ mho/m}$ , and  $\epsilon = .65 \times 10^{-9} \text{ ohm-m}^2$ . Then  $P_{\text{ohmic}} = 150 \text{ watts}$  (for the two collectors combined), leaving  $P_{\text{viscous}} = 1150 \text{ watts}$ . Applying Equations (7a) and (7b) gives  $\delta = 1.9$ . However, if we use Equation (5) to compute  $\delta$ ,  $\delta = 19$ . Although Equations (5)-(7) assume a semi-infinite geometry and neglect circulating currents, the difference is unexpected. The cause of the difference is, more likely, uncertainty of the values for parameters  $f$ ,  $\epsilon$ ,  $\sigma$ , and, particularly,  $d$ . (For completeness, if  $\delta = 19$ , by Equation (5), then  $P_{\text{viscous}} = 14,448 \text{ watts}$  by Equation (7b).)

With the PWRLOSS program, the losses were:  $P_{\text{ohmic}} = 308 \text{ watts}$ ,  $P_{\text{viscous}} = 1070 \text{ watts}$ , for a total loss (not including machine resistive and bearing and windage losses) of 1378 watts, compared with the

experimentally obtained combined loss of 1300 watts. Values of  $\delta$  varied between 3 and 10.

One other PWRLOSS run was made with this geometry. This second run included in the loop equations effects of  $J_x$  on vertical edges and  $J_y$  on horizontal edges (see question 7, previous section). The power losses each increased by 5%.

#### ACKNOWLEDGMENTS

The authors would like to acknowledge the assistance of Erwin A. Schroeder and Donald A. Gignac, both of Code 1844, Carderock Laboratory. In many fruitful discussions, Mr. Schroeder aided in reaching a better understanding of the physical problem, and Mr. Gignac gave many helpful hints towards solving the numerical problems.

#### REFERENCES

1. Carr, S.L., "Liquid Metal Current Collector Development Progress Report on In-House Investigations," Report DTNSRDC/PAS-77/17 (Dec 1977).
2. Rhodenizer, R.L., "Development of Solid and/or Liquid Metal Collectors for Acyclic Machines: Final Report," General Electric Company Report S-70-1032 (Feb 1970).
3. Rhodenizer, R.L., "Development of Solid and/or Liquid Metal Current Collectors for Acyclic Machines - Final Report, Tasks 4 and 5," General Electric Company Report S-71-1110 (Sep 1971).
4. "The NASTRAN Theoretical Manual (Level 17.5)," NASA SP-221(05) (Dec 1978).
5. Johnson, J.L., G.T. Hummert, and A.R. Keeton, "Liquid Metal Current Collectors for Homopolar Machines," IEEE Transactions on Power Apparatus and Systems, Vol. PAS-95, No. 4 (Jul/Aug 1976).
6. Sears, F.W. and M.W. Zemansky, "University Physics," second ed., Addison-Wesley Publishing Co., Inc., Reading, MA (1955), p. 522.
7. Roll, L.B., "Computational Solution of Nonlinear Operator Equations," John Wiley & Sons, Inc., New York (1969).

8. Carr, S.L., M.J. Cannell, and H.O. Stevens, "Initial Test Operation with Braid Type One Liquid Metal Current Collectors in a Laboratory Model Superconductive Generator," Report DTNSRDC/PAS-79/21 (to be published).

## APPENDIX - DESCRIPTION OF THE PWRLOSS PROGRAM

The PWRLOSS computer program computes the circulating currents and ohmic and viscous losses using these currents in the liquid metal current collectors used in superconducting motors and generators. The main program controls the establishment of the open-core array Z and the first group of pointers. This array and the pointers are described at the end of this appendix. Control transfers to all principal subroutines are made from main program PWRLOSS. A brief description of each subroutine is given here.

### SUBROUTINE DESCRIPTIONS

#### NSFIL

NSFIL reads the data computed by NASTRAN. This information, kept on permanent file, includes the number of finite elements, grid point information, and the values of the transport current densities JX, JY.

#### SUPDAT

SUPDAT reads the remaining input data from cards. These include fluid properties and the remaining geometric properties, such as the size of the collector gap, collector radius, stator width, etc.

#### ELVAR

ELVAR assigns fluid or non-fluid status to the elements and counts the number of fluid elements (NFLE). From this point on, only NFLE elements are used in computations.

#### VARYJ

From the transport current densities JX, JY for each element, as given by NASTRAN, VARYJ computes the values at mid-edges by averaging these densities with those of adjacent elements. The pointers for these values are IXT, IXR, IXB, IXL, IYT, IYR, IYB, IYL, corresponding to the top, right, bottom and left edges.

#### ZJ

This subroutine controls the major computations. Its primary functions are:

1. To assign values to remaining pointers of open core array Z.
2. To call CARRAY, which establishes initial values for circulating currents and the open core indices for them.

3. To set up a set of simultaneous equations, for the circulating current, referred to as  $i$ . The derivation of the equations and a solution to them are described below.

(3a) Each finite element can be considered an electrical circuit, or loop. Thus Kirchhoff's loop rule can be applied: The algebraic sum of the electromotive forces in any loop of a network equals the algebraic sum of the  $Ri$  products in the same loop, i.e.,  $\sum EMF = \sum Ri$ .

(3b) The induced electromotance

$$\oint \vec{E} \cdot d\vec{\ell} = \oint (\vec{V}_f \times \vec{B}) \cdot d\vec{\ell}, \text{ for } \vec{\ell}$$

around the loop, is computed. Therefore,  $\sum_j \hat{i}_j - \oint (\vec{V}_f \times \vec{B}) \cdot d\vec{\ell} = 0$  for each of the NFLE elements, where the summation is taken over the edges of the element, and  $\hat{i}_j$  is the sum of an unknown circulating current  $i_j$  and known transport current  $I_j$ .

(3c) The Kirchhoff point rule, that is, the sum of the currents at a point zero, is also applied at each grid point. There are NCUR unique circulating currents,

$$NCUR = 4 + 3(MROW + MCOL - 2) + 2 * (NFLE + 1 - MROW - MCOL),$$

where MROW and MCOL are the number of element rows and columns in the NaK fluid, respectively. The unknown variable  $\hat{i}_j$  in (3a), (3b), and (3c), on the element edges, is the sum of the unknown circulating current  $i$  and the known transport current  $I$ .

(3d) The NCUR circulating currents in the set of NCUR equations are described in (3a), (3b), and (3c). The set is designated by  $P$ . Because the velocity  $V_f$  will contain the expression  $\sqrt{2\delta - 1}$  for  $\delta > 1$ , where  $\delta$  is a function of  $i$ , the equations are, in general, non-linear. The Newton-Raphson method is used to linearize the equations and solve them by the Newton iterative process. A short description is given here, for the set of equations  $P(i) = 0$ .

Although the set  $P$  is nonlinear, it is differentiable. This leads to linearization by differentiation<sup>7</sup> and solution for  $\{i\}$  by the iterative Newton-Raphson method:

$P(i) = P(i_0) + P'(i_0)(i-i_0)$  leads to

$$P'(n) \begin{Bmatrix} \Delta i_1 \\ \vdots \\ \Delta i_n \end{Bmatrix} = -P^{(n)}, \text{ with each iteration yielding}$$

$\{i^{(n+1)}\} = \{i^{(n)}\} + \{\Delta i\}^{(n)}$ . Iterations continue until convergence is reached for

$$\left| \frac{i_{\ell}^{(n+1)} - i_{\ell}^{(n)}}{i_{\ell}^{(n)}} \right|_{\max} \leq .1$$

(3e) In subroutine ZJ the system  $P\{i\}^n$  is evaluated for a given set of circulating currents and stored at Z(ICC). The partial derivative with respect to  $j$ ,  $j=1,n$ , of this system  $P$ , is also evaluated, with results stored in Z(IPCC).

(3f) Command is turned over to PSOLV for the actual solution for  $\Delta i$  and the convergence test.

(3g) After convergence is reached or after a preset number of iterations has been made, control returns to PWRLOSS for summary.

#### CARRAY

Although there are NFLE fluid elements, each with four sides, many sides are shared by adjacent elements. For example, the bottom edge of one element may be the top edge of an element in the next row. This common edge, however, is assumed to have one circulating current. Just as the finite elements are numbered, so are these currents; in this case the indexing is done by CARRAY. CARRAY stores the appropriate index number for each edge of the element, at Z(ITOP), Z(IRT), Z(IBOT), or Z(ILT), so that the index of Z(IRT) of one element would be equal to that of Z(ILT) of an adjacent element.

The values of the circulating currents themselves are stored at Z(IC). The initial value of these currents is also computed in CARRAY. Those in the y-direction are assigned the average of the y-transport currents of the fluid elements, as given by NASTRAN. A similar

assignment is done for the x-direction.

#### ELPOS

Subroutine ELPOS assigns a position locator to each side of an element. These locators place an element edge:

- on the stator radial surface (4)
- on the left sidewall of the stator (5)
- on the right sidewall of the stator (6)
- in the radial gap (1)
- on the rotor tip (7)
- in axial gap (8)
- in axial gap or adjacent to left side rotor (3)
- in axial gap or adjacent to right side rotor (2)

These values are given to each of the output variables (for the element I):

- IVFT: position value for top of element
- IVFR: position value for right of element
- IVFB: position value for bottom of element
- IVFL: position value for left of element

These locators are used to determine what form of the velocity equation is to be used for that element.

#### PSOLV

PSOLV performs several functions.

- (a) It rearranges the arrays ICC (P) and IPCC (P') for use in IMSL subroutine LEQTLF.
- (b) From the solution of  $P'\Delta i = -P$ , as computed by LEQTLF, PSOLV tests for convergence and adds  $\Delta i$  to  $i$  for a new set of circulating currents.
- (c) Control returns to ZJ.

#### SUMARY

SUMARY computes  $\delta$  (Equation (8)) and fluid velocity (Equations (1) and (2)) using the circulating currents from ZJ. The  $\delta$ -values are computed from Equation (8) for each element edge.  $V_f$  is computed on each edge from the appropriate Equations (1)-(4), depending on  $\delta$ -value and direction of  $\vec{J} \times \vec{B}$ . The average of the four edge velocities for an element is the value actually used in computing viscous losses.



#### POHM

POHM computes the ohmic losses from Equation (9).

#### PVISC

PVISC computes the viscous losses, using the circulating currents and fluid velocities of ZJ and SUMARY, from Equation (10).

PWRLOSS uses the open-core concept to provide a variable-sized finite element mesh. The program itself requires a central memory size of 60000 octal words. To this must be added the length of the open-core, blank common array Z, whose size is determined by the formula

$$NCUR = 4 + 3(MROW + MCOL - 2) + 2 * (NFLE + 1 - MROW - MCOL).$$

The user must provide the number of finite elements (NE) and the number of fluid elements (NFLE), the number of rows (MROW) in NFLE, and the number of columns in NFLE (MCOL) or the number of circulating currents (NCUR). The number of words required is (in decimal)

$$31 * NE + 3 * NCUR + 2 * NCUR * NCUR.$$

After converting this value to octal and adding it to the program length, the user enters the result on control card RFL: RFL,XXXXXX.

The following pointers are used in the Z-array. The length of the array associated with each pointer is also given.

#### Length NE

IZ	$\Delta Z$
IR	$\Delta Y$
IZC	X-centroids
IRC	Y-radius of element
IGX	gridpoints, X-coordinate
IGY	gridpoints, Y-coordinate
JX	transport current densities, X-direction
JY	transport current densities, Y-direction
JXS	temporary storage for JX
JYS	temporary storage for JY
INF	position locator of element
ID	$\delta$ , per element
IV	$V_f$ , per element
IPX	ohmic loss, X, per element

Length NE (cont.)

IPY	ohmic loss, Y, per element
IVS	viscous loss, per element
IJX	average of circulating current X, per element
IJY	average of circulating current Y, per element
IXL	average X transport current density, left edge element
IXR	average X transport current density, right edge element
IXT	average X transport current density, top edge element
IXB	average X transport current density, bottom edge element
IYL	average Y transport current density, left edge element
IYR	average Y transport current density, right edge element
IYT	average Y transport current density, top edge element
IYL	average Y transport current density, bottom edge element

Length NCUR

IC	circulating currents
IDC	$\Delta i$ , returned from LEQT1F
ICC	P array, values of loop and EMF equations

Length NCUR\*NCUR

IPCC	P' array, partial derivatives of P
IFP	work space for LEQT1F

Length NFLE

IDD	final $\delta$ value, per element
IVV	final $V_f$ value, per element

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